Bank Runs and Inequality

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Abstract

I propose a banking model that provides a rationale for why increasing income inequality correlates with financial crises by looking at the probability of bank runs, which are recognized as triggers for financial crises. The canonical model of bank runs of Diamond and Dybvig (1983) is extended to accommodate heterogeneity in endowment levels between two groups of agents by having a mean-preserving distribution of endowments between groups. I find that, in equilibrium, the likelihood of a bank run does increase with income inequality. I corroborate this finding by analyzing data for 17 countries between 1880 and 2013, where I find that even accounting for macroeconomic variables, an increased probability of a bank run is correlated with an increase in the income share held by the top percentiles of the income distribution. The findings of the paper have important implications for policymakers, suggesting that reducing income inequality can help prevent financial crises.

1 Introduction

Financial crises have, among others, adverse effects on consumption and output, investment, productivity, employment, and health (Chodorow-Reich, 2014, Cutler et al., 2002, Jensen and Johannesen, 2017, Romer and Romer, 2017). Kirschenmann et al. (2016) and Paul (2022) identify income inequality as a significant predictor of financial crises in developed countries. I propose a model that provides a rationale for why increasing income inequality correlates with financial crises by looking at the probability of bank runs, which Friedman and Schwartz (1963) recognized as triggers for financial crises. Furthermore, I empirically document that income inequality is positively correlated with bank runs.

This article studies another possible mechanism that may affect the probability of a bank run. Specifically, this paper looks at how increasing income inequality affects this probability. The bank

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run model proposed by Diamond and Dybvig (1983) and Allen and Gale (1998) is extended to accommodate heterogeneity in endowment levels between two groups of agents by having a mean-preserving distribution of endowment between groups. The wedge between both groups describes the income inequality of the economy. This model considers only fundamental bank runs, where bank runs are the result of weakening fundamentals and thus cannot be avoided, in contrast to those produced by sunspots or self-fulfilling prophecies as coordinating mechanisms.

Furthermore, while it incorporates the assumptions on preferences and timing from Diamond and Dybvig (1983), it separates from their model in two ways: (i) it does not assume sequential withdrawal from the depositors, and (ii) the illiquid assets held by the bank have different productivity between the states of the economy. In my model, depositor preferences exhibit a decreasing relative risk aversion, which allows for different risk profiles between groups of depositors with differing endowment levels.[1]

The mechanism that mediates the relationship between income inequality and bank runs is as follows. Banks play a key role in protecting against liquidity shocks and aligning the demand for consumption allocations from deposit contracts with the preferences for asset portfolios (Allen and Gale, 1998, Diamond and Dybvig, 1983, Farhi et al., 2009, Goldstein and Pauzner, 2005). By allowing ex ante differences in the endowment level, the bank faces a challenge: it has to provide deposit contracts that are incentive compatible (i.e., those that induce truth telling about the depositors liquidity shocks) and satisfy the participation constraint (i.e., when the depositors are willing to take up the deposit contracts), while income inequality changes the risk profile of the depositors and the business cycle may lead to the productivity of the portfolio not being enough to provide the resources needed to cover for these deposit contracts.

Specifically, income inequality affects the agents' risk profile through a change in their relative risk aversion. In this case, less wealthy depositors become more risk averse than wealthier groups, making the former more willing to pool risk than the latter (Ogaki and Zhang, 2001). Agents face liquidity shocks and have preferences about their investment portfolio that satisfy their consumption

[1] This is consistent with Ogaki and Zhang (2001) that found that relative risk aversion varies across the income distribution. This type of preference also allows for increased risk sharing. These types of preferences are part of those represented by hyperbolic absolute risk aversion (HARA), which includes those with constant and increasing relative risk aversion. Additionally, this type of preference is within the Stone-Geary family of utility functions that are cardinal in nature. See Appendix A for a quick review of these preferences.

demands. As there is a single bank in this economy, the bank has to address the different liquidity shocks and risk profiles of both groups of depositors by offering deposit contracts. However, as income inequality increases, (a) the payment offered by the deposit contracts increase for the wealthy group as their outside option increases, while the opposite occurs to the less wealthy, and (b) the wealthier group becomes less risk averse, which in turn, allows them to absorb the increasing risk aversion of the less wealthy. There exists a threshold of income inequality such that the payments offered by the deposit contract are too large for the wealthy in order for them to participate and too low for the less wealthy in order to have non-negative consumption. After this threshold, the bank finds it optimal to offer deposit contracts that induce a bank run since these deposit contracts maximize the depositors surplus of the economy.

The equilibrium conditions for the case where there is no bank run by any depositor group provide the intuition behind the challenge for the bank. In the first place, holding all else equal, each group of depositors will have a different investment preference to satisfy their own trade-off between consuming in $t = 1$ or $t = 2$, before their type is revealed. However, there is a single portfolio decision for the bank, given the level of income inequality, which is common knowledge. This trade-off puts the bank in a situation where it weighs less the role of portfolio manager and weighs more the role of liquidity insurer. In the second place, holding all else equal, the consumption allocations depend on a function of the multipliers of the participation constraints. This function is increasing in income inequality, and this affects differently the consumption allocations between groups. As income inequality increases, consumption allocations (i.e., those for the early type and for the late type depending on the business cycle) for the wealthy increase, while they decrease for the less wealthy. This has a clear effect on relative risk aversion for both groups. It decreases risk aversion for the wealthy and increases for the less wealthy. In addition, it has a clear effect on how the bank will allocate the available resources from its asset portfolio. The bank will need to allocate more resources towards the deposit contracts of the wealthier group such that (a) it is willing to absorb the risk from the less wealthy group and (b) it takes on the deposit contracts.

The equilibrium conditions for the case where both depositors run a bank in the low state of the economy can also provide a sense of why the bank finds it optimal to induce the bank run. First, the bank will divide the available resources of their investments as a proportion of the first-period payment in case of no run. This is what I will call the proportionality of the decision. Since the participation constraints are the same in nature, that is, they are increasing income inequality for the wealthy group and the opposite for the less wealthy; the deposit contracts behave similarly as in the case where there is no run. Thus, the proportionality of the decision places more weight on the wealthy group than on the less wealthy group. Second, in equilibrium, consumption allocations must satisfy that the expected value of the difference in the marginal utility of consumption between groups of depositors in $t = 1$ must be equal to the expected present value of the difference in the marginal utility of consumption between groups in $t = 2$. As mentioned above, the consumption allocations are greater for the wealthy than for the less wealthy. However, in this case, the expected available resources are smaller since the fire sale rate of the risky asset is smaller than the return on the risky asset in the event of no bank run. Now, since the marginal utility is diminishing on consumption allocations, the only way that the equilibrium condition holds is that the consumption allocations for the less wealthy group are close to zero whenever the low state of the economy is realized. Since the less wealthy participation constraint is decreasing with inequality, the bank is always able to induce the participation of this group even with smaller equilibrium allocations and to induce the participation of the wealthy with larger consumption allocations.

There is a possibility of intermediate cases. The first one is where the wealthy group does not run, and the less wealthy group runs. The second intermediate case is the exact opposite, where the wealthy group runs and the less wealthy group does not run. More importantly, the equilibrium conditions are a combination of those present in the case with no bank run and those present in the case of a bank run. In these particular cases, the bank faces the following restriction: if the sum of promised deposit contracts at $t = 1$ is smaller than the available resources in case of a run, then it is possible to promise contracts for the group that does not run in the low state. In equilibrium, these two intermediate cases will be dominated by either of the first two cases.

In sum, the bank will select the consumption allocations from either case whenever the optimal depositor surplus is greater. Since I am focusing on fundamental bank runs, in the event of equality between the case where no depositor runs and the other cases, the former will always be selected instead of the latter. In this sense, there exists a level of income inequality after which the optimal surplus of the cases where there is a run is greater than that of the case without a run, holding all else equal. Thus, the probability of a bank run happening in equilibrium is increasing in income inequality.

I solved the model numerically and found that in equilibrium, the probability of a bank run does increase with income inequality. The bank's decision to allow this run stems from the fact that it is optimal to offer deposit contracts that induce a bank run. Furthermore, the consumption allocations of the wealthier group increase with increasing inequality, whereas the opposite occurs for the less wealthy group. There are two main reasons behind this result. First, by allowing the preferences to display decreasing relative risk aversion, a change in inequality modifies the preferences for deposit contracts that fit the changing risk profile of each group of agents. This would not have been possible with standard Constant Relative Risk Aversion (CRRA) preferences. Second, the main results can be attributed to the fact that increasing inequality raises the outside option for the former while it decreases for the latter. Looking at the investment portfolio, the bank must balance it toward illiquid assets when inequality increases. Once the income inequality reaches a point where a run is likely to occur, the bank adjusts the risk profile of the portfolio: it reduces the amount of illiquid assets while still maintaining a balanced approach to that type of asset. These changes in the riskiness of the portfolio are consistent with the changing risk preferences of depositors reflected in their new consumption demands for deposit contracts.

The main finding of the model was corroborated by examining the statistical correlation between income inequality and bank runs. The analysis suggests a positive correlation between increased income inequality and the likelihood of bank runs. A standard deviation increase in income inequality is associated with a 0.3 to 0.7 percentage point increase in the probability of a bank run. Since the unconditional probability of a bank run was estimated to be around 4% per year, a percentage point increase is a significant increase.[2]

Literature Review. The literature has recently focused on the determinants of financial crises (see Baron et al. (2021), Gorton and Ordoñez (2020), Kirschenmann et al. (2016), Paul (2022)). Researchers have focused on a wider set of financial crises, following the definitions of Laeven and Valencia (2012), Schularick and Taylor (2012), and Reinhart and Rogoff (2009), among others. From the vast set of determinants, Kirschenmann et al. (2016) and Paul (2022) have found that income inequality has predictive power for financial crises, in a general sense, in developed countries. My contribution in this area is to document the particular correlation between income inequality (as measured by Paul (2022)) and bank runs (as defined by Baron et al. (2021)).[3]

[2] This result holds for different covariates. A big caveat to this analysis is that with the available data it is not possible to determine causality (that is, an increase in income inequality precedes a bank run).

[3] Baron et al. (2021) do not explicitly define bank runs as fundamental bank runs, they define them as bank panics. Thus, it is not possible to determine whether they are fundamental bank runs ex-ante.

The literature has also examined the theoretical channel between income inequality and financial fragility. Malinen (2016) provides a brief but interesting review of this channel, documenting that the relationship between income inequality and financial crises operates through the bank credit channel: An increase in income inequality leads to a rise in bank credit or leverage, amplifying credit cycles (which, in turn, can generate bank runs and then financial crisis). For example, Kumhof et al. (2016) finds that increasing income inequality leads to the accumulation of debt-to-income ratios, which results in a financial crisis. In summary, there is a relationship between income inequality and credit cycles in which credit accumulation plays a fundamental role. My contribution is to establish a channel between income inequality and fundamental bank runs.

This paper contributes to the literature on income inequality and financial fragility (see Choi (2014) and Mitkov (2020)). More importantly, Garcia and Panetti (2022) looks into a context similar to that in this paper. They investigated how wealth inequality makes financial crises more likely, finding that higher wealth inequality directly increases the incentives to run for the poor and indirectly for the rich through higher bank liquidity insurance. These incentives make bank runs more likely to be self-fulfilling. To achieve these results, they made use of two main assumptions. First, they have multiple balance sheets that ring-fence the asset investment by wealth level, acting as universal banks. Second, they have an investment externality assumption that accounts for the spread of wealth between wealth groups, ultimately leading to bank runs. The main difference in my contribution is that even in a model with a unique balance sheet and no investment externality, fundamental runs will occur if the inequality is large enough.

Finally, this paper contributes to the literature on modeling bank runs from the seminal work of Diamond and Dybvig (1983). More precisely, it extends the model presented in Allen and Gale (1998) to accommodate ex ante identical agents with different endowment levels to account for inequality.

This paper is divided as follows. Sections two, three, and four present the model, the numerical exercise, and the results and discussion of such results, respectively. Section five presents the correlation between income inequality and bank runs in the data. Section six concludes.

2 A Model of Income Inequality and Bank Runs

In this section, I elaborate on the banking model.

2.1 Preliminaries, Preferences, and Endowments

There are three periods indexed to $t = 0, 1, 2$. Two possible states of nature $s = H, L$ occur with probability π_H and π_L such that $\pi_H + \pi_L = 1$. The state of nature becomes common knowledge at $t = 1$. There is a continuum of agents with mass two composed of two groups of equal mass that differ only in their initial level of endowments. The two groups are indexed as $i = 1, 2$. Each depositor within a group is indexed by j. Without loss of generality, let the group of depositors $i = 1$ be endowed with ω_1 units of the final consumption good and let the depositors in the group $i = 2$ be endowed with ω_2 units, where $\omega_1 > \omega_2 > 0$. These endowment levels are common knowledge. Depositors receive the endowment at $t = 0$ and do not receive any additional endowment at $t = 1, 2$. However, they want to consume $t = 1$ or $t = 2$, depending on the realization of a liquidity shock.

These depositors are subject to a liquidity shock. This shock is i.i.d across the depositors. That is, they are uncertain about the timing of their consumption. If the depositor j can only consume at $t = 1$, he is of type early, while if he only consumes at $t = 2$, he is of type late. These types are not common knowledge, but let the probability of being of the early type be $\lambda \in (0, 1)$, and, consequently, the probability of being of the late type be $1 - \lambda$, known to all agents. Given the equal mass of groups and the law of large numbers, the parameter λ can be interpreted as the proportion of agents that are of the early type. The depositor j will know its type at $t = 1$.

The typical depositor j of the group i has preferences represented by a utility function $U(c_{ti})$ that is increasing, strictly concave, and differentiable twice continuously. Let the utility function be

$$U(c) = \frac{(c - \psi)^{1-\gamma}}{1 - \gamma} \tag{1}$$

This utility function in (1) represents Hyperbolic Absolute Risk Aversion (HARA) preferences. More importantly, let $\psi \geq 0$ and $\gamma > 0$ so that (1) exhibits decreased relative risk aversion (DRRA).[45] Note that with this functional form of utility, the consumer will only have positive utility whenever $c > \psi$. The depositor will enjoy utility after a minimum consumption allocation is provided, also known as a subsistence level of consumption.[6]

[4]Note that this particular utility function belongs to the family of Stone-Geary utility functions. More importantly, this family of utility functions is cardinal in nature, rather than ordinal in nature. Thus, the sign of this particular utility function is important.

[5]See Appendix A for a discussion of HARA preferences.

[6]The only restriction on the consumption allocations is that they should be different from ψ. This is to have the Relative Risk Aversion coefficient always defined. Although the allocation of consumption under these preferences

In terms of information available to the depositor, the depositor j does not know if they are of type early or late until $t = 1$. He also does not know what would happen in $t = 2$. Define consumption allocations as d_i for those early consumers in $t = 1$ and c_{2is} for those late consumers in $t = 2$ for all $i = 1, 2$ and $s = H, L$. Therefore, let the expected utility of the typical depositor j in group i be described as follows:

$$u\left(d_i, c_{2is}\right) = \sum_{s=H,L} \pi_s \left[\lambda U\left(d_i\right) + (1-\lambda)\beta U\left(c_{2is}\right)\right] \tag{2}$$

where $\beta > 0$ is a common discount factor for both groups. Note that to truly reveal his type, the incentive compatibility constraint for agent j in group i is given by:

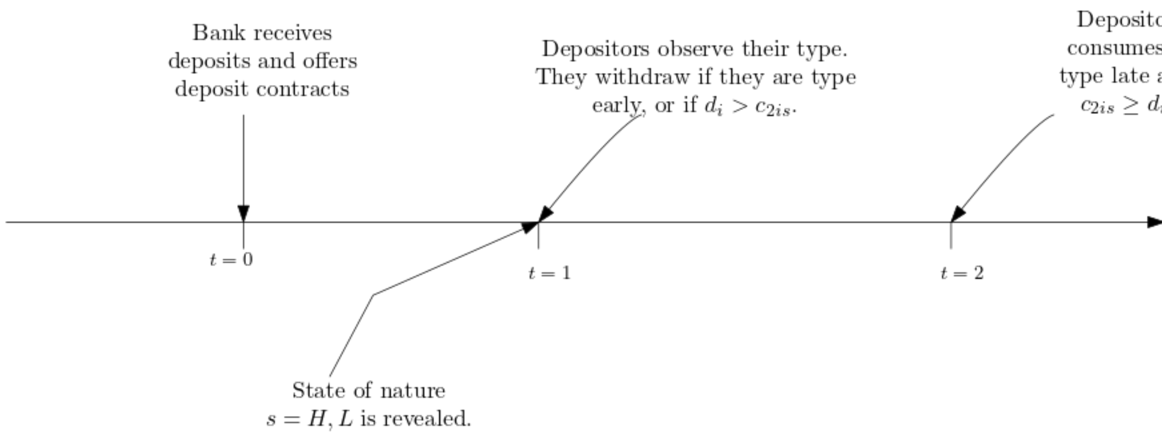
$$d_i \leq c_{2is} \quad \forall i = 1, 2, s = H, L \tag{3}$$

Finally, to ensure that the depositor accepts the deposit contract offered by the bank, this contract has to satisfy the participation constraint in the form of

$$\sum_{s=H,L} \pi_s \left[\lambda U\left(d_i\right) + (1-\lambda)\beta U\left(c_{2is}\right)\right] \geq U\left(\omega_i\right) \text{ for } i = 1, 2 \tag{4}$$

The timing of the problem is presented in Figure 1. The bank receives the deposits at $t = 0$ and offers the deposit contracts at the end of that period. Most of the action occurs at $t = 1$. In this period, the depositor's type is revealed to the depositor (but not to the bank), and the state of nature is revealed to all economic agents. Then, if the incentive compatibility constraints hold, the depositors will withdraw at their respective periods, and there will be no run. However, a run will occur if the incentive compatibility constraint is not satisfied.

Figure 1: The Timing of the Problem



can be 0, the parameter ψ induces a situation of disutility whenever the allocation is 0. Thus, this allocation will be dominated by any allocation greater than 0.

2.2 Banks's Portfolio

There is a bank that takes the endowments of the depositors $\omega_1$and ω_2, and invests them in a portfolio made up of:

- A liquid asset (short-term) y with a constant return to scale technology that takes one unit of consumption good at t and transforms it into one unit of consumption good at $t+1$ for $t = 0, 1$. This technology can be thought of as a storage technology. Agents also have access to this technology.

- An illiquid and risky asset x that has a constant return to scale technology that takes one unit of the consumption good in $t = 0$ and transforms it into R_H units of the consumption good with probability $\pi_H \in (0, 1)$ in $t = 2$ or into R_L units of the consumption good with probability $\pi_L \in (0, 1)$ in $t = 2$, where $\pi_H + \pi_L = 1$ and $R_H > R_L > 1$. Therefore, there are two possible states of nature $s = H, L$. In the early liquidation of this asset, the technology takes one unit of consumption good at $t = 0$ and transforms it into $r \in (0, 1)$ units of consumption good at $t = 1$.

Introducing this random asset return does not rule out bank runs that occur out of self-fulfilling prophecies or sunspots as a coordination mechanism. Thus, I am considering only essential bank runs (that is, bank runs that cannot be avoided). That is, in the case that there exist multiple equilibria in which one of the possible equilibria there is no run and the others happen to induce a bank run, I will select that with no bank run.

Once the bank receives the endowments from the depositors at $t = 0$, it has to choose an investment portfolio (x, y) such that

$$x + y \leq \omega_1 + \omega_2 \tag{5}$$

This is a feasibility constraint for the bank. It suggests that the entire portfolio should be less than or equal to the total endowments in the economy. Now, let $\omega_1 \equiv 1 + \tau$ and $\omega_2 \equiv 1 - \tau$ for a $\tau \in (0, 1)$ such that it complies with the assumption that $\omega_1 > \omega_2$ without increasing the size of the economy, that is, without making $\omega_1 + \omega_2$ greater. Then, (5) becomes

$$x + y \leq 2 \equiv \omega \tag{6}$$

Note that the larger τ is, the greater inequality becomes. I set τ to move freely between 0 and 1.

As the bank can only purchase assets with the aggregate level of endowment in the economy, it is not ring-fencing its services to attend a specific wealth group.

2.3 Bank's Problem

There is free entry and competition among banks in this economy, so they maximize the surplus of depositors. More importantly, this implies that the bank will have zero profits in equilibrium. The depositors' surplus given by

$$W\left(d_1, d_2, c_{21H}, c_{22H}, c_{21L}, c_{22H}, y, x\right) = \sum_{i=1,2} \sum_{s=H,L} \pi_s \left[\lambda U\left(d_i\right) + (1-\lambda)\beta U\left(c_{2is}\right)\right] \tag{7}$$

First, the bank can only purchase the assets with the endowments from the depositors. This condition is captured in (6).

Second, there is a proportion λ that will be of early type, and the bank has to acquire enough liquid assets to provide for the deposit contracts d_1 and d_2. That is,

$$\lambda\left(d_1 + d_2\right) \leq y \tag{8}$$

Third, the bank's zero-profit condition leads to the depositors receiving all of the remaining value of the assets at $t = 2$. Then, the bank faces the following constraints:

$$(1-\lambda)\left(c_{21s} + c_{22s}\right) = R_s x + y - \lambda\left(d_1 + d_2\right) \quad \forall s = H, L \tag{9}$$

The bank also needs to provide deposit contracts that are incentive compatible (3) in order to prevent a bank run, but it could also provide contracts that does not satisfy such condition and induce a run. Finally, the bank always need to satisfy the participation constraint of the depositor such that the depositor takes the deposit contract. Thus, the bank also faces the constraints in (4).

In summary, the bank's maximization problem in the case where there is no bank run is given by[7]

$$\max_{\{d_i\}_{i=1,2}, \{c_{2is}\}_{i=1,2}^{s=H,L}, y, x} W\left(d_1, d_2, c_{21H}, c_{22H}, c_{21L}, c_{22H}, y, x\right)$$

[7] The bank's maximization problem will be similar in nature as that where it induces a bank run. I will present this case more in the detail in the corresponding subsection.

subject to

$$
\begin{aligned}
x + y &\leq 2 \equiv \omega \\
\lambda \left(d_1 + d_2\right) &\leq y \\
(1-\lambda)\left(c_{21s} + c_{22s}\right) &= R_s x + y - \lambda \left(d_1 + d_2\right) \quad \forall s = H, L \\
d_i &\leq c_{2is} \quad \forall i = 1, 2, s = H, L \\
U\left(\omega_i\right) &\leq \lambda U(d_i) + (1-\lambda)\beta \left[\pi_H U(c_{2iH}) + \pi_L U(c_{2iL})\right] \quad \forall i = 1, 2
\end{aligned}
$$

In the next section, I will discuss the possible cases the bank will face depending on how the incentive compatibility conditions hold. The bank's problem will be a variation of the problem described above and will be discussed in each corresponding subsection.

2.4 Bank's Possible Cases

Now, the bank cannot know at $t = 0$ which state of nature will occur at $t = 2$. The state of nature is revealed at $t = 1$ at the same time the deposit contracts are being offered. The late type depositor in the group $i = 1, 2$ can run or not in the bank depending on whether the incentive compatibility constraint is satisfied or not. Therefore, the bank could potentially face up to 10 cases involving different maximization problems. Table 1 summarizes the possible cases.

Table 1: Possible Cases if Group $i = 1, 2$ Runs

State	**None**	**Both**	**Group 1**	**Group 2**
Low	Case 1	Case 2	Case 3	Case 4
High	Case 1	Case 5	Case 6	Case 7
Both	Case 1	Case 8	Case 9	Case 10

However, some cases are not optimal and can be discarded beforehand. The following propositions are aimed at discarding some non-optimal cases.

Proposition 2.1. *There will never be a fundamental run in both states $s = H, L$ for both groups of agents.*

Proof. See Appendix B.1 ☐

The intuition behind this is that the contract that induces a fundamental run in both states for both groups of agents is dominated by a contract that offers at least the same amount as the previous contract in $t = 1$ and a positive amount in $t = 2$ because the return in either state $s = H, L$ is greater than 1. The social utility of the second contract is greater than that of the original contract. According to this proposition, Case 8, where the agents are running in both states, is not optimal.

Proposition 2.2. *It is never optimal to choose a contract that leads to a run in the high state for some group i but does not lead to a run for that group i in the low state.*

Proof. See Appendix B.2 □

The intuition is that this type of contract opens the possibility of a residual of the amount distributed at $t = 2$ that allows for a welfare-improving allocation where there is no run in the high state. It does not necessarily lead to a run in the low state for the group i. According to this proposition, the cases where groups 1 - case 6 - or 2 - case 7 - run in the high state and not in the low state cannot be optimal.

Proposition 2.3. *It is never optimal to choose a contract that leads to a run in* ***BOTH*** *states $s = H, L$ by some group i.*

Proof. See Appendix B.3. □

The intuition is that a contract that leads to a run in both states by a group i is dominated by a contract where the group $j \neq i$ is at least as good as before and the group i is strictly better. This proposition suggests cases where group one, case 9 or group 2, case 10 cannot be optimal. Furthermore, the case where both groups run in the high state cannot be optimal by extension of Propositions 2.2 and 2.3.

The remaining cases are case 1 (no group runs in any state), case 2 (both groups run in low state), case 3 (group one runs in low state), and case 4 (group 2 runs in low state).

2.4.1 Case 1: No agents run in both states

In the first case, neither agent runs in the bank in either or both states. Let d_i for $i = 1, 2$ be the face value of the deposit contract at $t = 1$. The bank maximization problem is given by:

$$\max_{d_1,d_2,y,\{c_{2is}\}_{i=1,2,s=H,L}} \sum_{s=L,H} \pi_s \left\{\lambda \left[U\left(d_1\right) + U\left(d_2\right)\right] + (1-\lambda)\,\beta\left[U\left(c_{21s}\right) + U\left(c_{22s}\right)\right]\right\} \tag{10}$$

subject to

$$\lambda\left(d_1 + d_2\right) \leq y \tag{11}$$

$$(1-\lambda)\left(c_{21s} + c_{22s}\right) = R_s\left(\omega - y\right) + y - \lambda\left(d_1 + d_2\right) \quad \forall s = H, L \tag{12}$$

$$c_{2is} \geq d_i \quad \forall i = 1, 2, s = H, L \tag{13}$$

$$\lambda U(d_i) + (1-\lambda)\beta\left[\pi_H U(c_{2iH}) + \pi_L U(c_{2iL})\right] \geq U\left(\omega_i\right) \quad \forall i = 1, 2 \tag{14}$$

$$0 \leq y \leq \omega \tag{15}$$

Condition (11) suggests that the total face value of the deposit contracts offered to the λ share of early-type depositors in both groups should be less or equal to the amount invested in the liquid asset y. More importantly, Proposition 2.4 implies that this condition should be in equality, since it is never optimal to leave some investment in the liquid asset from $t = 1$ to $t = 2$ because if the same amount were invested in the illiquid asset, it would yield $R_s > 1$ units more at $t = 2$.

Proposition 2.4. *The condition $\lambda\left(d_1 + d_2\right) \leq y$ should hold with strict equality in optimum.*

Proof. See Appendix B.4 □

The condition (12) holds with equality since the bank will give back all of what is available to the depositors at $t = 2$, in any state $s = H, L$. By the result on the proposition (2.4), this condition can be rewritten as

$$(1-\lambda)\left(c_{21s} + c_{22s}\right) = R_s\left(\omega - y\right) \tag{16}$$

Condition in (13) is the incentive compatibility constraint for both agents in both states if the bank offers a contract that leads to no runs by either agent in both states. These conditions imply that, to induce truth telling from the agents to reveal their type, those of the late type are provided with a consumption allocation greater than that of the early type.

The condition (14) is the participation constraint of both agents. These conditions imply that for the agent to take the deposit contract offered by the bank, the expected value of that contract should exceed the utility of consuming their endowment.

The condition (15) is a bank feasibility constraint that implies that the investment in the liquid asset should be greater than or equal to 0 and less than the total endowment of the economy. This condition will hold with strict inequality due to the non-negativity constraints of consumption allocations.

Equilibrium Conditions for Case 1

This problem has three decision moments. In the first, the bank has to decide the level of investment y and x. In the second moment, the bank offers the deposit contracts at $t = 1$ conditional on the invested portfolio. The last moment corresponds to the depositor's decision to run or not on the bank conditional on the deposit contracts offered by the bank and the revelation of private information at $t = 1$. In this section, I focus on the equilibrium conditions in the second moment, which is conditional on the level of investment y. Also, note that case 1 is based on the depositor's decision to not run on the bank.

Let δ_4 and δ_5 be the multipliers of the constraints in (14), and define $A \equiv U'^{-1}\left(\frac{1+\delta_5}{1+\delta_4}\right)$. From the first order conditions of the problem defined by (10) - (14), I found the following consumption allocations were found as a function of y and A.

$$d_1 = \frac{Ay}{\lambda(1+A)} \tag{17}$$

$$d_2 = \frac{y}{\lambda(1+A)} \tag{18}$$

$$c_{21L} = \frac{AR_L(\omega - y)}{(1-\lambda)(1+A)} \tag{19}$$

$$c_{22L} = \frac{R_L(\omega - y)}{(1-\lambda)(1+A)} \tag{20}$$

$$c_{21H} = \frac{AR_H(\omega - y)}{(1-\lambda)(1+A)} \tag{21}$$

$$c_{22H} = \frac{R_H(\omega - y)}{(1-\lambda)(1+A)} \tag{22}$$

First note that the allocations for the depositors are increasing in y at $t = 1$ while decreasing in y at $t = 2$. This is not more than the trade-off of the bank faces in order to insure for the liquidity shocks of the depositors, since the productivity of the illiquid asset is greater than the 1-to-1 return of the liquid asset (that is, $R_H > R_L > 1$). In case 1, the bank does not need to worry about the liquidation value $r < 1$.

Second, in the case that none of the participation constraints is ever binding (that is, $\delta_4 = \delta_5$), it means that A becomes $U'^{-1}\left(\frac{1+\delta_5}{1+\delta_4}\right) = 1 + \psi$, given the utility function in (1). This means that A increases in $\psi \geq 0$, and for depositors in groups 1 and 2 they will increase and decrease in ψ, respectively. However, increasing ψ implies the need for greater consumption allocations to obtain positive utility outside the subsistence level. In this sense, as ψ increases, depositors in group 2 face the situation that, for a given level of investment y, their allocations are decreasing and it may be that their utility is 0 or negative.

Third, let $\delta_4, \delta_5 > 0$ and $\delta_4 \neq \delta_5$, that is let the participation constraints be binding and the multipliers on that constraint be different between depositors.[8] This suggests that $A = \left(\frac{1+\delta_4}{1+\delta_5}\right)^{\frac{1}{\gamma}} + \psi$. Holding all else constant, A is increasing in δ_4 and decreasing in δ_5. How would each participation constraint behave under this scenario? Note that for group 1, the participation constraint is given by

$$U(\omega_1) = \lambda U\left(\frac{Ay}{\lambda(1+A)}\right) + (1-\lambda)\beta\left[\pi_H U\left(\frac{AR_H(\omega - y)}{(1-\lambda)(1+A)}\right) + \pi_L U\left(\frac{AR_L(\omega - y)}{(1-\lambda)(1+A)}\right)\right] \quad (23)$$

The left hand side of this expression is increasing on τ since $\omega_1 = 1 + \tau$. Holding all else constant, the participation constraint becomes more straining as inequality increases (i.e., the depositor of group 1 becomes richer). This means that δ_4 needs to increase to make A larger to compensate for the increase in inequality. Thus, δ_4 increases in τ.

For the depositor group 2, the participation constraint is given by

$$U(\omega_2) = \lambda U\left(\frac{y}{\lambda(1+A)}\right) + (1-\lambda)\beta\left[\pi_H U\left(\frac{R_H(\omega - y)}{(1-\lambda)(1+A)}\right) + \pi_L U\left(\frac{R_L(\omega - y)}{(1-\lambda)(1+A)}\right)\right] \quad (24)$$

On the contrary, the left hand side of this expression is decreasing on τ since $\omega_2 = 1 - \tau$. This suggests that, all else constant, δ_5 must be increasing to make A smaller to compensate for the increase in inequality. Thus, δ_5 increases in τ. In total, with both δ_4 and δ_5 increasing in τ, one can assume that A increases in income inequality, holding everything else equal.

Furthermore, let A^* be the A that solves the equation system in (23) and (24), which is a function

[8] Note that in the case that $\delta_4 = 0$ and $\delta_5 = 0$ the same intuition holds as in discussed with $\delta_4 = \delta_5$ but with the participation being binding.

of y and τ. Consumption allocations for group 1 increase in τ since they increase in A^*, while the opposite is true for group 2, given the level of investment y.

Fourth, note that the Euler equation for each i is described by

$$U'(d_i) = \beta \left(R_H \pi_H U'(c_{2iH}) + R_L \pi_L U'(c_{2iL}) \right) \quad \forall i = 1, 2 \tag{25}$$

From this expression, and the result for A^*, one can infer that for group 1 the marginal cost of consuming at $t = 1$ and the expected marginal benefit from consuming at $t = 2$ is increasing as income inequality increases, while the opposite occurs to group 2. This can be explained by their change in the respective relative risk aversion (RRA), given by

$$RRA = \frac{-cU''(c)}{U'(c)} = \frac{\gamma c}{c - \psi} \tag{26}$$

For group 1, the consumption allocations are increasing in A, the RRA is decreasing (i.e., $\frac{\partial RRA}{\partial c} = \frac{-\psi\gamma}{(c-\psi)^2} < 0$ and $\psi \geq 0$), holding all else equal. The opposite occurs with the depositors of group 2, where consumption allocations are decreasing in A, which in turn, RRA is increasing. This suggests that as income inequality increases, depositors in group 1 are more willing to take on more risk than those in group 2. The bank then has to balance this change in the risk profile of both depositor groups with the change in the outside option of each group to calculate consumption allocations that do not induce a bank run, given a level of investment y.

Finally, the bank's decision on y will be that that solves (25). Given the characteristics of the current problem, it is not possible to provide an analytic solution given the non-linear relationship in terms of y in (25). However, the level of investment y required to satisfy the Euler condition for group 1 will be different from that of group 2, but the bank will only choose one level of investment y. To illustrate this point, define $z_t = c_t - \psi$, so that one can approximate the solution of y using a log-linearization of (25) around z^* for both groups. The y_i^* for $i = 1, 2$ from such procedure is a local solution but can provide some insight.[9] The solutions are

$$y_1^* = \frac{\lambda \bar{R} \omega}{\lambda \bar{R} + (1 - \lambda)} + \frac{\lambda(1-\lambda)(1+A)}{\lambda \bar{R} + (1-\lambda)} \left(\log(\beta) + E\left[\log(R)\right] \right) \tag{27}$$

$$y_2^* = \frac{\lambda \bar{R} \omega}{\lambda \bar{R} + (1 - \lambda)} + \frac{\lambda(1-\lambda)(1+A)}{A\left(\lambda \bar{R} + (1-\lambda)\right)} \left(\log(\beta) + E\left[\log(R)\right] \right) \tag{28}$$

where $\bar{R} \equiv \pi_H R_H + \pi_L R_L$. Figure 2 presents the different solutions for y_i^* as a function of A for a set of given parameters. The investment decision for group 1 increases in A, while for group 2

[9]The linearization process is described in Appendix C

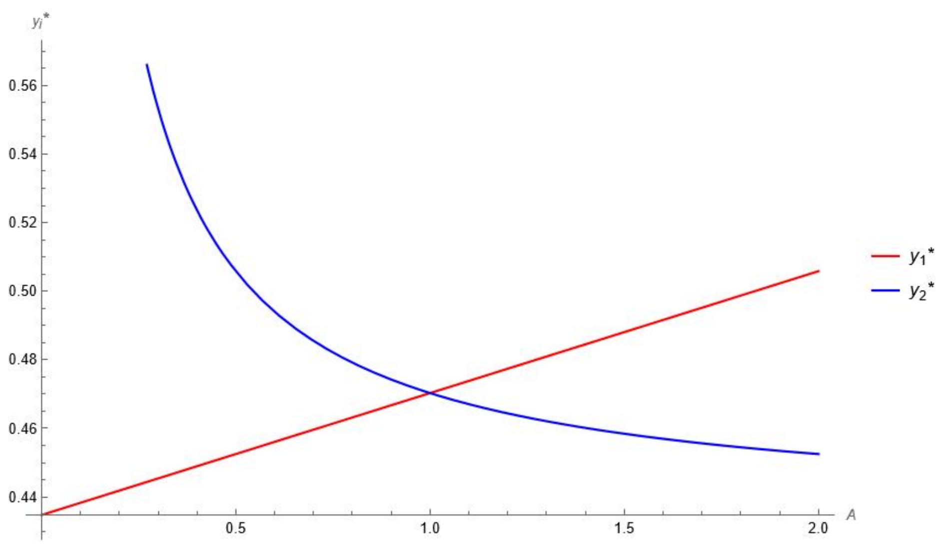
it decreases and intersects only once at $A = 1$, which is consistent with the egalitarian allocation where the allocations are split equally between the depositor groups. That is,

$$d_i = \frac{y}{2\lambda} \quad \forall i = 1, 2 \tag{29}$$

$$c_{2is} = \frac{R_s(\omega - y)}{2(1-\lambda)} \quad \forall i = 1, 2, s = H, L \tag{30}$$

The previous discussion is based on a local solution for y. It may be the case that y^* can still be unique in the nonlinear solution of the equation system in (25). However, the previous discussion still suggests that the bank needs to account for the portfolio preferences of both groups in order to provide the deposit contracts that do not induce a bank run.

Figure 2: Linearized solution for y_i^* as a function of A



2.4.2 Case 2: Bank Offers a Contract such that All Run in the Low State

The second case is that both agents run on the bank in the low state. Let $\tilde{d}_{is}$ be the consumption allocation received by the depositors in case of a bank run in state L. The bank's maximization problem is given by

$$\max_{d_1, d_2, y, \{c_{2is}\}_{i=1,2}^{s=H,L}} \pi_H \sum_{i=1,2} \{\lambda [U(d_i) + (1-\lambda) U(c_{2iH})]\} + \pi_L \left\{U\left(\tilde{d}_{1L}\right) + U\left(\tilde{d}_{2L}\right)\right\} \tag{31}$$

subject to

$$d_1 + d_2 \geq [r(\omega - y) + y] \tag{32}$$

$$c_{2iH} \geq d_i \quad \forall i = 1, 2 \tag{33}$$

$$\tilde{d}_{iL} = \frac{d_i}{d_1 + d_2} [r(\omega - y) + y] \quad \forall i = 1, 2 \tag{34}$$

$$\lambda (d_1 + d_2) = y \tag{35}$$

$$(1 - \lambda)(c_{21H} + c_{22H}) = R_H (\omega - y) \tag{36}$$

$$\pi_H \{\lambda U(d_i) + (1 - \lambda) U(c_{2iH})\} + \pi_L \left\{U\left(\tilde{d}_{is}\right)\right\} \geq U(\omega_i) \quad \forall i = 1, 2 \tag{37}$$

$$0 \leq y \leq \omega \tag{38}$$

In this case, the portion for state $s = H$ is the same since there is no run. However, for $s = L$, it changes to accommodate the fact that both agents are running in this state. In this case, the entire mass of both groups of agents is running. Thus, this portion does not depend on λ. In terms of the restrictions, first note that conditions (33), (35), (36) and (38) are similar to that in the case where no agent runs.

The incentive compatibility condition in state $s = L$ (32) requires some additional explanation. This condition is to avoid unilateral deviation of the late-type depositor. In this case, the deviation is that the late depositor does not run and waits to consume at $t = 2$. In case of a run, the bank has to liquidate their long-term asset at a fire sale rate of $r < 1$ and use it, in addition to whatever the bank has on the liquid asset, to pay for the consumption allocations of the agents that run. This is captured by the expression on the right-hand side of the condition (32).

Suppose that condition (32) is violated so that the amount of liquidated assets is greater than the deposit contracts in $t = 1$. In this case, the bank can pay the deposit contracts to all agents that ran at their face value at $t = 1$. Given the assumption that the bank has to return whatever is left in $t = 2$, the agent that deviated would receive a large amount (infinity) in $t = 2$. Since all agents of type late $(1 - \lambda)$ are ex-ante identical, all agents of type late would be incentivized to deviate and wait until $t = 2$ for consumption. In this case, a welfare-improving allocation would not be to liquidate the illiquid assets since $R_L > r$ and all agents would have been better off, and this case would not occur. It follows that to have a run from both groups at $s = H$ condition (32) should always be satisfied.

The condition (34) suggests how the liquidated value of the assets is distributed in the event of a run. In this case, it is distributed proportionally to the face value of the deposit contracts promised by the bank. Finally, the participation constraint (37) has the same intention as in the case of no run, but is modified to accommodate run allocation at $s = L$.

Equilibrium Conditions for Case 2

The problem described in (31)-(38) and the economic setup of this problem involve a series of nonlinearities that make it impossible to solve analytically for the equilibrium allocations and investment y. However, there are certain equilibrium conditions that can shed some light on the trade-off faced by agents in this economy. In the first place, the consumption allocations for $s = H$ are described by a similar condition to that of Case 1. That is,

$$c_{21s} = \frac{AR_s(\omega - y)}{(1-\lambda)(1+A)} \quad \forall s = H, L \tag{39}$$

$$c_{22s} = \frac{R_s(\omega - y)}{(1-\lambda)(1+A)} \quad \forall s = H, L \tag{40}$$

where A is still defined as before $A \equiv U'^{-1}\left(\frac{1+\delta_5}{1+\delta_4}\right)$. Using the first-order conditions, I can get the following equilibrium condition:

$$\pi_H \left(U'(d_1) - U'(d_2)\right) + \pi_L \left(U'(\tilde{d}_1)\frac{\tilde{d}_2}{y} - U'(\tilde{d}_2)\frac{\tilde{d}_1}{y}\right) = R_H \pi_H \beta \left(U'(c_{21H}) - U'(c_{22H})\right) \tag{41}$$

The right-hand side of (41) is the present value of the difference in the marginal utility of the consumption allocations in $t = 2$ that occur with probability π_H. As mentioned above, the consumption allocations $t = 2$ behave similarly as in the previous case: increase (decrease) in A for group 1 (2) and decrease both in y. So, for any level of income inequality τ, $c_{21H} > c_{22H}$, thus $U'(c_{22H}) > U'(c_{21H})$, and this difference in marginal utility is positive.

The left-hand side of (41) has two terms. The first term is the difference between the marginal utility of consumption in state H that occurs with probability π_H. This difference will be positive as long as $d_2 > d_1$, and negative otherwise. Note that the term $\frac{\tilde{d}_i}{y}$ for $i = 1, 2$ captures how much of the allocation given to agent i at $t = 1$ in case of a bank run is accounted for in each unit of liquid asset y. Also, the term $\frac{\tilde{d}_i}{y}$ is multiplying the marginal utility of agent j with $i \neq j$. Therefore, the marginal utility of the allocation of consumption to the depositor i is valued in terms of the allocation per unit of liquid asset given to the depositor j, with $i \neq j$.

2.4.3 Cases 3 and 4: Bank Offers A Contract Where One Group Runs and the Other Does Not in the Low State

First, note that Cases 3 and 4 are identical with the changed subscripts of the group of agents. Then, let group 2 be the one that runs on the bank, and group 1 does not run to the bank (that is, case 9). The bank's maximization problem is given by:

$$\max_{d_1,d_2,y,\{c_{2is}\}_{i=1,2}^{s=H,L}} \pi_H \sum_{i=1,2} \{\lambda [U(d_i)(1-\lambda)\beta U(c_{2iH})]\} + \pi_L \left\{U\left(\tilde{d}_{2L}\right) + \lambda U\left(\tilde{d}_{1L}\right) + (1-\lambda)\beta U(c_{21L})\right\} \quad (42)$$

subject to

$$\begin{cases} c_{21L} \geq \tilde{d}_{1L} & \text{IC rich in low state} \\ \tilde{d}_{2L} \geq c_{22L} & \text{IC poor in low state} \end{cases} \quad (43)$$

$$c_{2iH} \geq d_i, i = 1,2 \quad (44)$$

$$\lambda (d_1 + d_2) = y \quad (45)$$

$$\tilde{d}_{2L} = \begin{cases} d_2 & \text{If } \lambda d_1 + d_2 \leq r(\omega - y) + y \\ \frac{d_2}{\lambda d_1 + d_2} [r(\omega - y) + y] & \text{otherwise} \end{cases} \quad (46)$$

$$\tilde{d}_{1L} = \begin{cases} d_1 & \text{If } \lambda d_1 + d_2 \leq r(\omega - y) + y \\ \frac{\lambda d_1}{\lambda d_1 + d_2} [r(\omega - y) + y] & \text{otherwise} \end{cases} \quad (47)$$

$$(1-\lambda)(c_{21H} + c_{22H}) = R_H(\omega - y) + y - \lambda(d_1 + d_2) \quad (48)$$

$$(1-\lambda) c_{21L} = \begin{cases} 0 & \text{if } \lambda d_1 + d_2 > r(\omega - y) + y \\ R_L \left\{\omega - y - \left[\frac{\lambda d_1 + d_2 - y}{r}\right]_+\right\} + [y - (\lambda d_1 + d_2)]_+ & \text{otherwise} \end{cases} \quad (49)$$

$$\pi_H \{\lambda U(d_1) + (1-\lambda) U(c_{21H})\} + \pi_L \left\{\lambda U\left(\tilde{d}_{1L}\right) + (1-\lambda) U(c_{21L})\right\} \geq U(\omega_1) \quad (50)$$

$$\pi_H \{\lambda U(d_2) + (1-\lambda) U(c_{22H})\} + \pi_L \left\{U\left(\tilde{d}_{2L}\right)\right\} \geq U(\omega_2) \quad (51)$$

$$0 \leq y \leq \omega \quad (52)$$

where $[x]_+ = \max\{x, 0\}$

The first portion of the objective function is the sum of utilities if the state $s = L$ occurs. The second portion is the sum of the utilities in the state $s = L$. In this case, a proportion of agents

$(1-\lambda)$ of group 1 will not run on the bank, while the entire mass of group 2 will run. Conditions (44), (45), (48), (50), and (51) are similar to those of the other cases.

The conditions in (43) are the incentive compatibility constraints for both groups in the state $s = L$. In this case, the incentive compatibility constraint for group 1 aims to deter the deviation of the late type to run on the bank, while for group 2 the objective is to deter the deviation of the late type to consume at $t = 2$ since, in this case, the bank offers a contract that induces a run in the low state for this group. Focus on the incentive compatibility for Group 2. In this situation, the bank must liquidate the illiquid asset in a fire sale at a rate $r < 1$ (in addition to the liquid asset) and use this to pay the consumption allocations promised in $t = 1$ for all the types early and late that run on the bank. That is, the bank has to pay $r(\omega - y) + y$.

The condition in (49) presents the availability of resources from which the proportion $(1-\lambda)$ of late depositors in group 1 will receive their deposit contract. The availability will depend on the value of the deposit contracts paid to the depositors who withdraw their consumption allocations in $t = 1$. If the value of such deposits is greater than the available resources from the early liquidation of the risky asset in addition to the amount of liquid asset, then there will be no resources left to pay the $1-\lambda$ share of late depositors. In contrast, the available resources will be the value of the risky asset that matured in $t = 2$.

Equilibrium Conditions for Cases 3 and 4

The problem described in (42)-(52) and the economic setup of this problem involve a series of non-linearities that make it impossible to solve analytically for the equilibrium allocations and investment y. However, one can assume that the equilibrium conditions will present a combination of those presented for Cases 1 and 2.

2.4.4 Bank's Case Selection in Equilibrium

Let $W_k(d_1^k, d_2^k, c_{21H}^k, c_{22H}^k, c_{21L}^k, c_{22L}^k)$ be the social utility valued in the optimal allocations in case $k = 1, 2, 3, 4$. The bank will then choose case k over all other cases $-k$ whenever the social utility of case k is at least as good as the maximum social utility of case $-k$. That is,

$$W_k(\cdot) > \max\{W_{-k}(\cdot)\} \wedge W_1(\cdot) \text{ if } W_1(\cdot) = \max\{W_j(\cdot)\} \quad \forall j = k/1 \tag{53}$$

where $-k$ all the other cases but the k^{th}. In case of equality, since I am focusing only on fundamental runs, I will let it resolve towards the case that does include a bank run.

3 Numerical Exercise

In this section, I present the results of a numerical exercise to demonstrate some of the properties of the model. The main goal is to look at how a change in τ, which implies changing the level of inequality, affects the various allocations of consumption, investment, the welfare function and, more importantly, the probability of a bank shutdown.

The parameters used in the numerical exercise are presented in Table 2. I set the parameter ψ at 0.4 so that the utility function presents a decreasing relative risk aversion. The parameters R_H and R_L imply that the risky asset pays 1.5 units of consumption goods per unit when it matures in the state H or 1.065 units in the state L. The parameter r implies that the recovery rate of the risky asset when liquidated early is about 80% the original investment value. I used $\gamma = 3$ as standard. I set the parameter λ, the share of early-type depositors, at 15%. Finally, I set the probability of the low state at 20%.

Table 2: Set of parameters for numerical exercise

Parameter	λ	τ	R_H	R_L	γ	ψ	r	π_L	β
Value	0.15	[0.30,0.90]	1.5	1.065	3.0	0.4	0.8	0.2	0.94

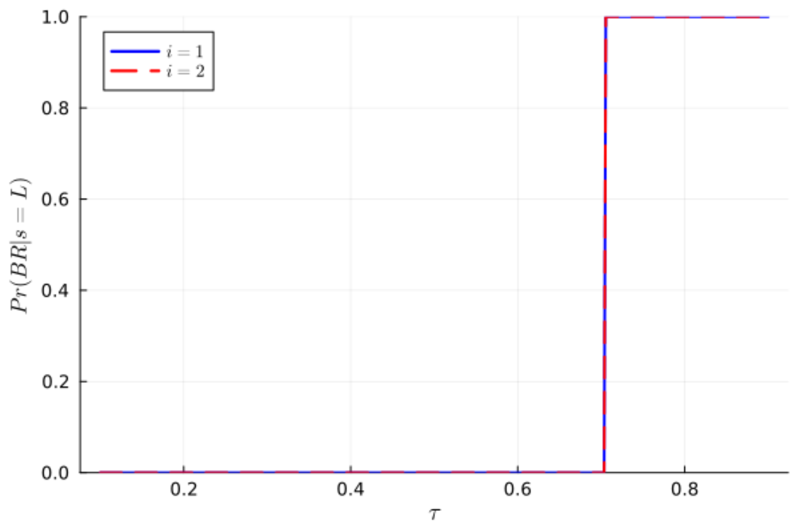
To estimate the model, I fixed the economic parameters and a level of τ and used the restrictions to bind grids of possible consumption allocations and investments. Given these consumption allocations and the investment level, the utility was estimated for each case. Then I proceeded to estimate the maximum utility given the fixed level of τ. Once I went through the entire grid of τ, I verified that the equilibrium conditions in (53) are satisfied by each level of τ.

4 Results and Discussion

4.1 Results

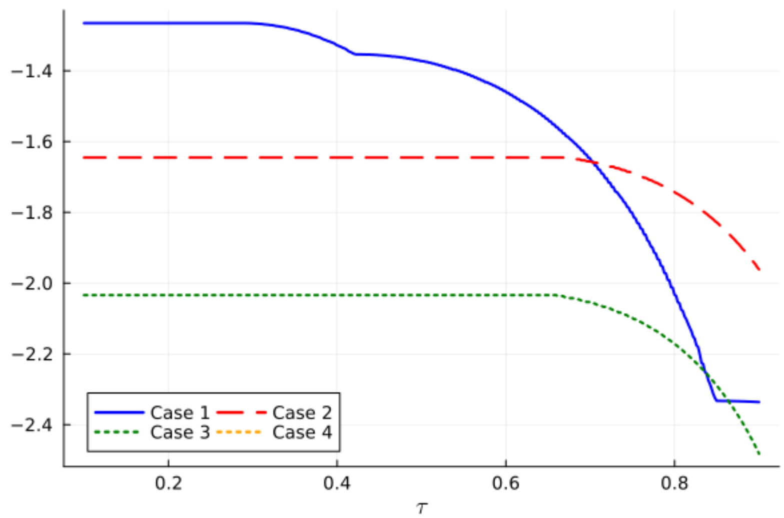
The main result of the numerical estimation of the model is that the probability of a bank run increases with the income inequality, measured by τ. This result is presented in Figure 3. Note that given the structure of the problem, runs can happen only in the low state. Thus, the probability of a bank run, conditional on being in the state L, is one once it reaches a sufficiently high τ (that is, τ^*). The unconditional probability is given by $Pr(BR|s = L) \times Pr(s = L) = \pi_L = 0.15$. The jump in the probability of a bank run is due to the discrete nature of the actions of either agent (that is, to run or not to run). This first result provides a possible rationale for the correlation found in the data.

Figure 3: Conditional Probability of Bank Run



Second, I discuss the social utility function $W_k(\cdot)$ for each case $k = 1, 2, 3, 4$, presented in Figure 4. Focusing only on cases 1 and 2, one can see that the social utility function of case 1 (no bank run) intersects with that of case 2 (bank run) at τ^*. Given that the upper contour of the set of social utility functions gives the largest social utility, one can affirm that after τ^* it is socially optimal for the bank to commit to consumption allocations that induce a bank run. Note that the social utility functions that arise from Cases 3 and 4 are never optimal for the set of parameters used. The reason behind this is as follows. Assume case 3, where group 2 is given an allocation that induces a run, and group one does not. Since there is a run, the bank has to liquidate all its illiquid assets at $t = 1$ with a fire sale rate $r < 1$. This leaves the bank with $s^* \equiv r(\omega - y) + y$ units of the final consumption good to allocate in the λd_1 and d_2 deposit contracts. Suppose that s^* is just enough to cover the deposit contracts at $t = 1$. This, in turn, leaves $(1 - \lambda)$ depositors of the late type of group one with 0 consumption at $t = 2$ because there is no illiquid asset available to mature at $t = 2$. Therefore, since $d_1 > 0 = C_{21L}$, the incentive compatibility constraint is not satisfied, and the late-type depositors in group one will run. A similar story will occur for Case 4.

Figure 4: Utility Functions for Cases 1, 2, 3, and 4



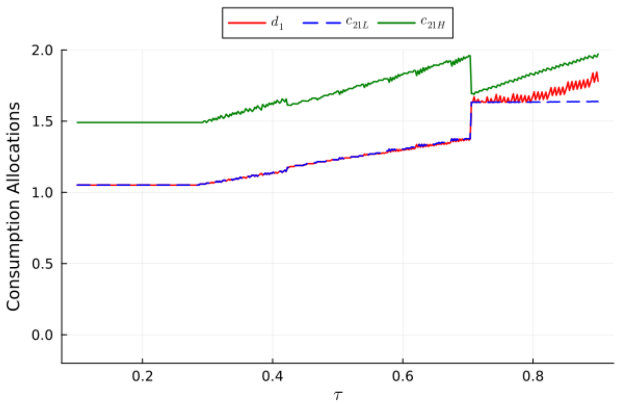
Third, I present the equilibrium consumption allocations for groups 1 and 2 in Figures 5a and 5b, respectively. Note that for group one (group 2), consumption allocations increase (decreasing) with τ. This trend in consumption allocations is because a higher income inequality (that is, an increase in τ) implies a larger outside option for group one, since their endowment increases with τ. On the contrary, the opposite happens for group 2. Then, the consumption allocation for each level of τ should be larger (smaller) for group 1 (group 2) to participate. Furthermore, depositors will only have positive utility when d_i and c_{2is} are greater than $-\psi$ for $i = 1, 2$ and $s = H, L$.[10] One can understand the parameter ψ as the minimum consumption required for subsistence.

In fourth place, the investment portfolio is presented in Figure 6. The portfolio must be balanced with the illiquid asset x to match the investment needs of the large proportion of late-type depositors in both groups. Once the level of inequality reaches τ^*, the portfolio must accommodate a lower risk by increasing the acquisition of liquid assets y. However, the total composition is still balanced toward the illiquid asset. This is because there is still the possibility of realizing the high state. Therefore, the bank must provide deposit contracts that cover this contingency while compensating for the loss of early liquidation in case the low state occurs, since the fire sale rate is $r < 1$.

[10]See footnote 4 for why under the utility function of this particular problem, the utility sign matters.

Figure 5: Equilibrium Consumption Allocations for Both Groups

(a) Group $i = 1$



(b) Group $i = 2$

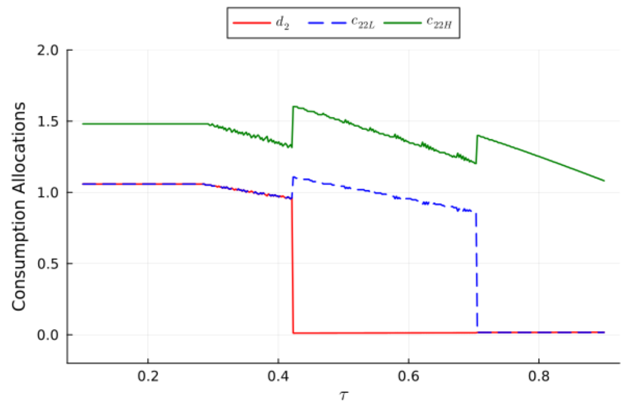
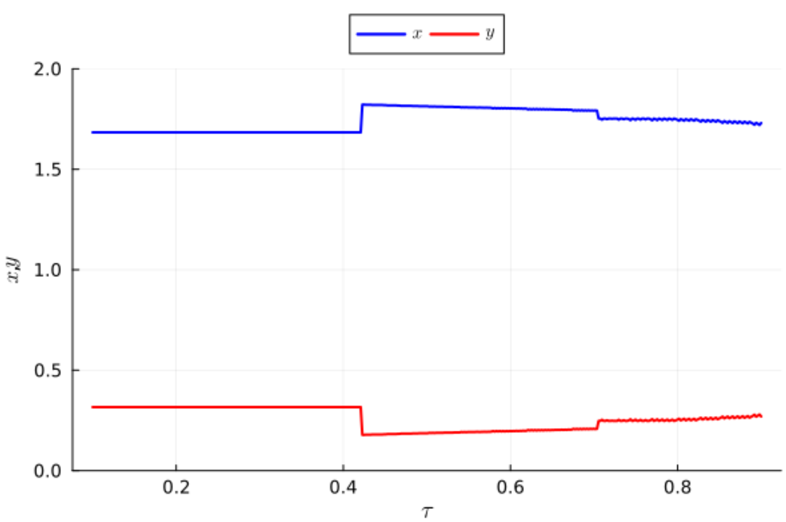


Figure 6: Investment Portfolio



4.2 Mechanism Discussion

What is the mechanism behind the result that the probability of a bank run increases with increasing income inequality? The answer to this question is that increasing income inequality, where group one sees its endowment increase and group two sees it decrease, triggers changes in their relative risk aversion affects the bank's role as liquidity insurer and portfolio manager, thus altering the financial stability of the economy.

Recall the discussion of the equilibrium conditions for Cases 1 and 2. First, note that the bank had only one investment decision for y and this is almost horizontal, suggesting that given the level of income inequality, the unique investment decision was almost uniform across τ. This result allows us to compare the consumption allocations discussed in that section, holding the investment level constant and all else equal. The presented that the consumption allocations increased in inequality for group 1 and decreased for group 2. The reason behind this result was the role of the participation constraint multipliers δ_4 and δ_5 that interacted in the function A the following way.

$$A = U'^{-1}\left(\frac{1+\delta_5}{1+\delta_4}\right) = \left(\frac{1+\delta_5}{1+\delta_4}\right)^{\frac{-1}{\gamma}} + \psi \tag{54}$$

It was also inferred that A was increasing in τ since it needed to expand or compress the participation constraint for depositors take up the deposit contracts. Therefore, as τ increases and the investment level is constant, the consumption allocations increase for group 1 and decrease for group 2 almost exclusively with changes in δ_4 and δ_5.

Note that the numerical exercise presents a clear pattern on the consumption allocations: they are increasing for group 1 and decreasing for group 2, and those for group 1 are always larger than those for group 2. Then, as τ increases, the bank must allocate more of the available resources to the deposit contracts of group 1.

At the same time, the risk aversion profile for both groups is changing. Group 1 is becoming less risk-averse as their consumption allocations increase. This is because their relative risk aversion is decreasing (that is, the larger the consumption allocation offered, the less risk averse they become). Thus, they are more willing to share risk, at least before τ is reached, that is, the probability of the bank run jumps to 1. This can be seen with the difference between d_1 and c_{21L} almost non-existent and the flat consumption profile for c_{22L}. This will happen until $c_{21L} < d_1$. This is the level of τ where depositors in group 1 will run in the bank and the consumption allocations for group 2 are too small (even lower than ψ). Then, the bank finds optimal (i.e. maximizes the depositors' surplus) to allow a bank run.

In conclusion, increasing income inequality drives changes in relative risk aversion, making it challenging for the bank to fulfill its role and inducing financial instability. Depositors exhibit changes in risk behavior between early and late types in both groups, with some becoming more risk averse, while others displaying less risk averse. The bank must limit its portfolio management strategies and instead offers deposit contracts favoring the participation of the richest group (i.e., higher deposit contracts) in this way the wealthier group is more willing to share the risk and allow deposit contracts that yield positive utility for the less wealthy group. This can be done until its role as a liquidity insurer is challenged by deposit contracts that inevitably lead to a bank run if the business cycle is bad.

4.3 Comparative Statics

The rest of this section presents different comparative statics involving changes in the return of the illiquid asset in the low state R_L, changes in the liquidity preferences captured by λ, and changes in the fire-sale rate r. These parameters are more likely to affect the probability of a run, all else equal, because of its direct incidence in the low state.

Changes in the return of the return of the illiquid asset in the low state are presented in Figures 7a and 7b. Note that the decrease or increase in return shifts τ^* to the left or right, respectively. In return, this change affects the size of the final total good available for distribution in state L. Thus, the increase in the return implies that the bank can pay for deposit contracts in cases with more inequality. The opposite occurs in the case where the return decreases.

Changes in liquidity preferences have big impacts on the probability of a run, all else equal. This result is presented in Figures 8a and 8b. Liquidity preferences are captured by the parameter λ, which is the probability that a given depositor is of the early type. Then the changes in λ reflect whether a depositor is willing to wait longer (less) to consume in $t = 2$. The numerical results of a small decrease in λ suggest that, all else equal, the probability of a run is 0 in the domain of τ (Figure 8a). A small increase in λ moves τ^* to the right. This is because increasing λ reduces the mass of late-type depositors; thus, all else equal, the bank will be more likely to fulfill the deposit contracts at higher levels of inequality.

Figure 7: Changes in $Pr(BR|s = L)$ with changes in R_L

(a) Decreasing R_L (b) Increasing R_L

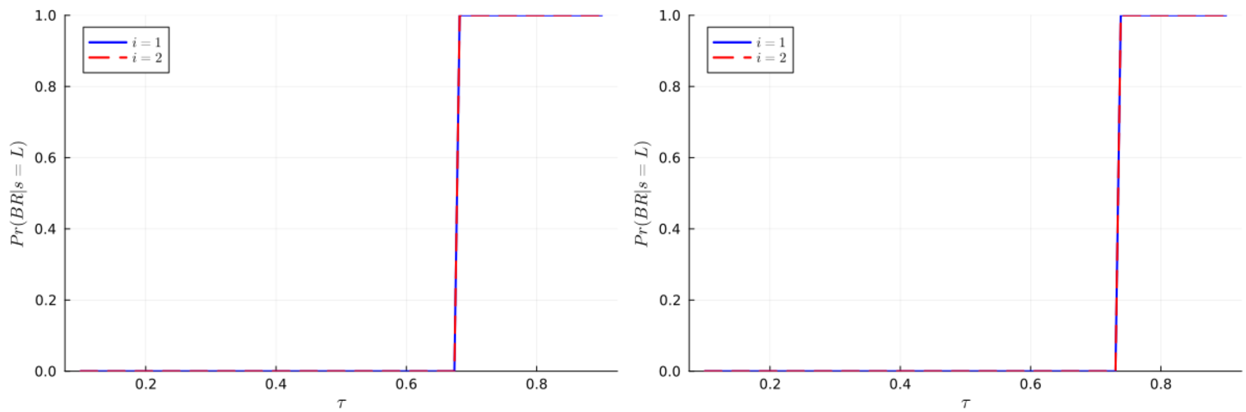
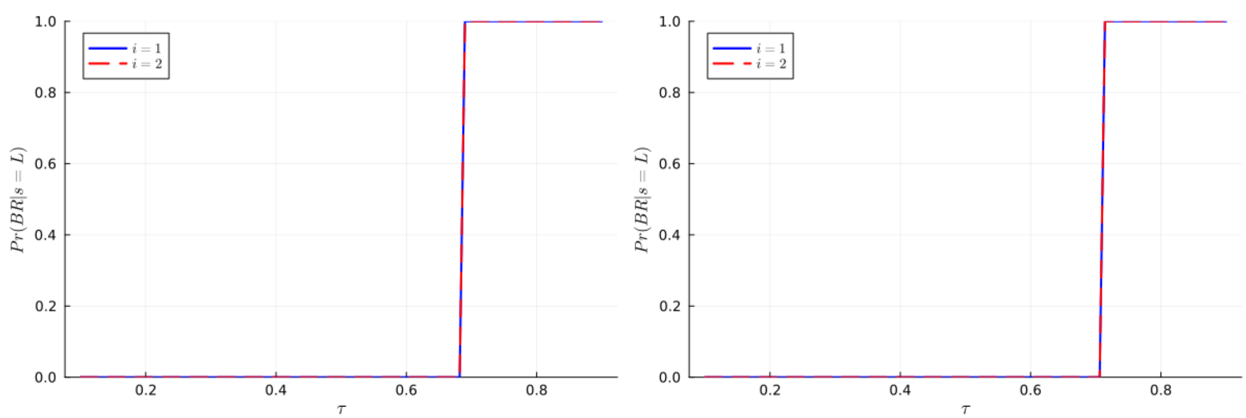


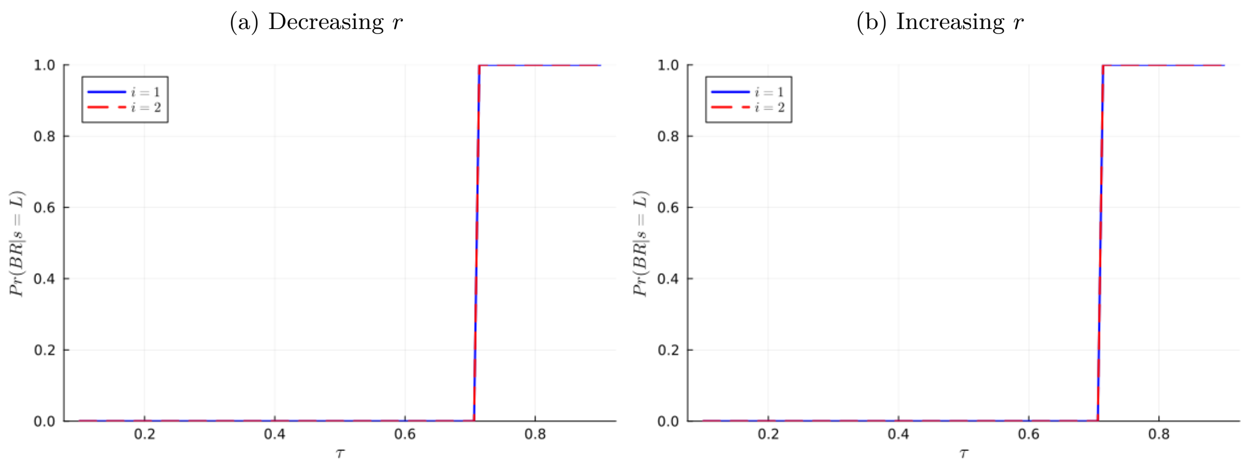
Figure 8: Changes in $Pr(BR|s = L)$ with changes in λ

(a) Small Decrease in λ (b) Small Increase in λ



Finally, changes in the fire-sale rate emerge as a significant determinant in the probability of a bank run within our model. Specifically, increasing the fire-sale rate increases the return when banks are forced to liquidate their assets. This return, in turn, increases the incentive to accept deposit contracts for potential depositors available on a run. Consequently, it increases the probability of a bank run.

Figure 9: Changes in $Pr(BR|s = L)$ with changes in r



5 The Correlation Between Bank Runs and Income Inequality

I use historical data in this section to discuss the correlation between income inequality and bank runs. First, I will describe the data used to investigate this correlation. Second, I present evidence on the prevalence of bank runs in advanced economies and how inequality behaves before such an event. Finally, I present estimations that shed light on the correlation between bank runs and income inequality.

5.1 Data

I used two novel data sets. First, I use the data compiled by Paul (2022) on income inequality. This data set merges three long-term data sets from 1870 to 2013 for 17 countries.[11] The first long-term data set is from Òscar Jordà et al. (2016), which includes macrofinancial variables for these 17 countries. The second long-term data set is from Bergeaud et al. (2016), which includes

[11]The countries are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

measures of TFP and labor productivity. The third long-term data set is the World Inequality Database, which includes measures of income shares held by various percentiles. The novel feature of the data set of Paul (2022) is that it includes income shares held by the upper percentiles, net of capital gains.[12]

In the second place, I use the data set for bank runs found in Baron et al. (2021). The authors collected information for 46 countries in a similar time frame as in Paul (2022). More importantly, they collect bank run narratives under a common definition to capture bank runs.[13]

The final data set includes information for 17 countries from 1880 to 2013, accounting for 2,069 country-year observations of macroeconomic, income inequality, and bank run variables.

5.2 The Prevalence of Bank Runs and Inequality Trends

Figure 10 presents the prevalence of bank runs for these 17 countries between 1880 and 2013. According to Allen and Gale (2007), bank runs are nothing new and have not been restricted to emerging economies. Baron et al. (2021) suggests almost no evidence of non-fundamental runs occurring in this time frame. More importantly, bank runs are distributed across the period except for the post-WWII years (i.e., 1945-1970). Using the data set, I estimate that the unconditional probability of bank runs is around 4% per year.

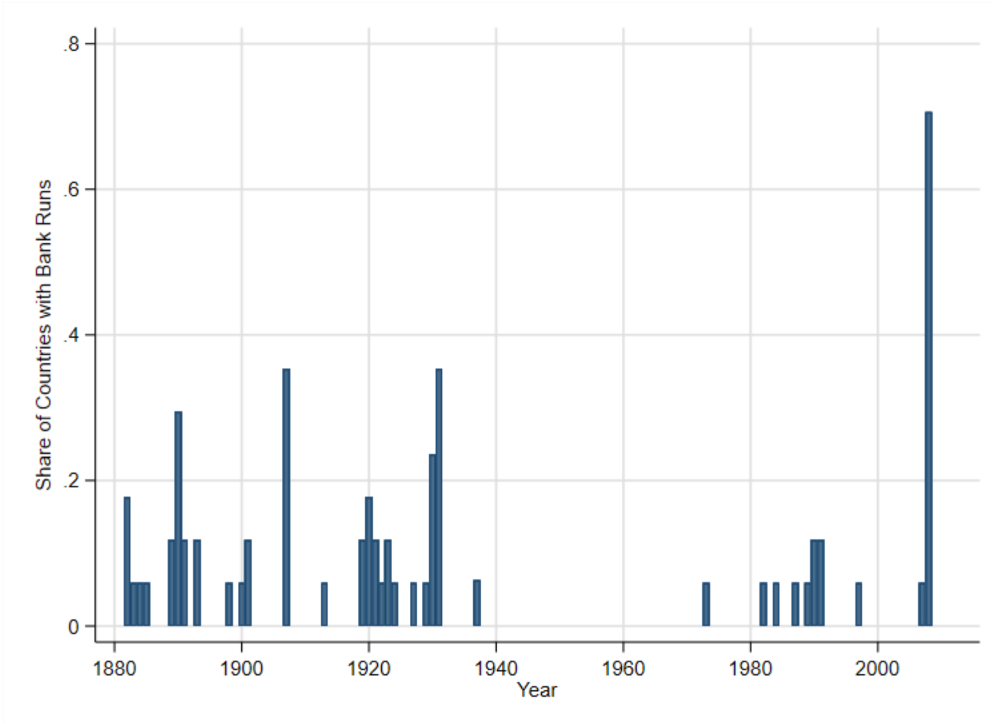
The measure of inequality that I use is the share of income held by the top 0. 1%, 1% and 10% of the income distribution net of capital gains found in Paul (2022). The trends of such inequality measures are presented in Figure 11. The dashed red line is the mean, while the solid blue line is the median. The gray areas represent the 33rd and 66th percentiles of the inequality measures in the countries in the sample.

First, note that the inequality trends presented in Figure 11 have long cycles (that is, they oscillate very slowly). For instance, they increased steadily from the late 19th century until the 1930s. Then, they decreased similarly in the post-WWII period until the 1980s, when they increased again until the end of the sample's time frame. Second, the sample median for each of the three

[12] The income share data is constructed using tax income data. The revenues of selling assets (i.e., stocks) are taxed depending on the tax system. The potential problem is that stock trading is typically concentrated among individuals in the upper percentile of the income distribution. This, in turn, makes capital gains available for reinvestment in the same assets, and the additional savings are not available for borrowing from other agents.

[13] Baron et al. (2021) describe bank runs as banking panics that are also bank equity crises (p. 102).

Figure 10: The Prevalence of Bank Runs



Source: Author estimation based on the data from Baron et al. (2021), Paul (2022).

Note: The sample includes 17 advanced-economy countries described in footnote 11.

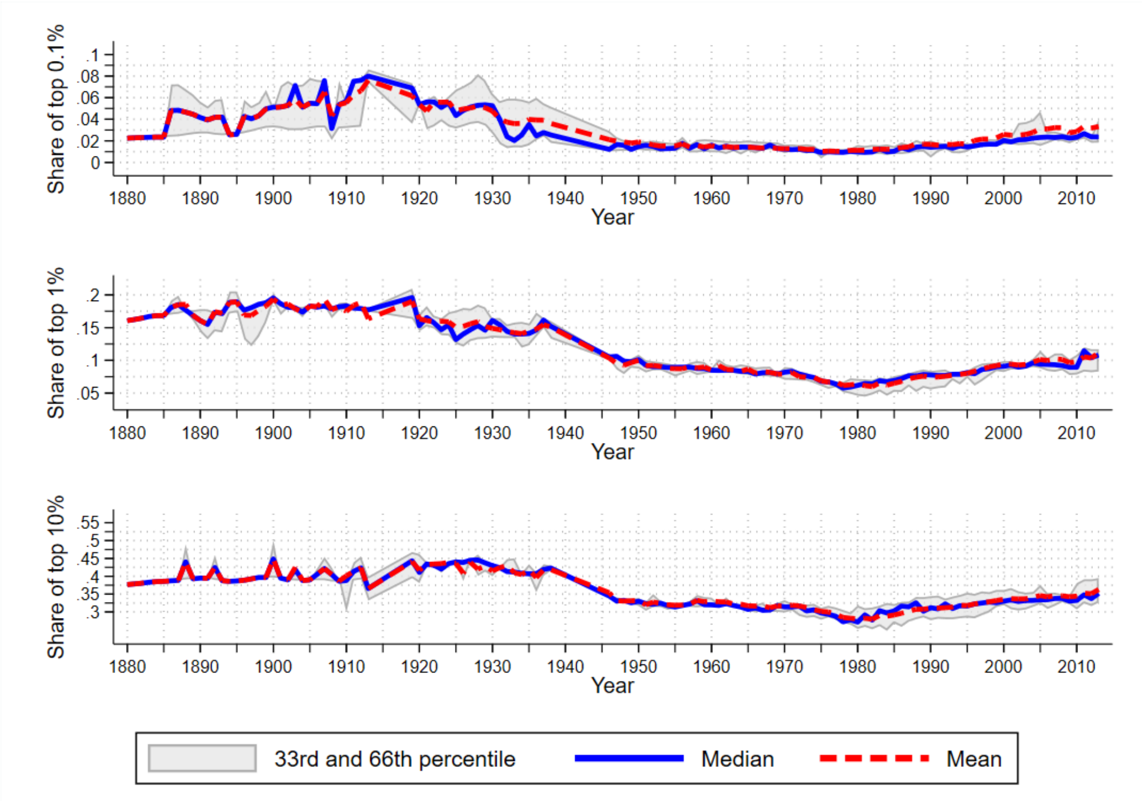
measures is 2.8%, 12.1%, and 35.7%, for the top 0.1%, 1%, and 10%, respectively. These suggest that between 1880 and 2013, in 50% of the countries in the sample, almost 35% of income was held by the top percentiles of the income distribution. Finally, the dispersion of such measures (i.e., the difference between the 33rd and 66th percentile) increases in increasing inequality, while it collapses in decreasing inequality.

These three features are by no means an exhaustive look at inequality trends around the world because they only apply to the countries in the sample. However, they account for the fact that inequality has increased since 1980 to the present day (see Piketty and Saez (2003)).

5.3 Estimating the Correlation Between Bank Runs and Income Inequality

Does changes in the level of income inequality precede the occurrence of a bank run? The answer to this question is empirical. To achieve this goal, I perform two analyses. In the first place, I perform a statistical analysis that resembles a naive event study to provide evidence on how the

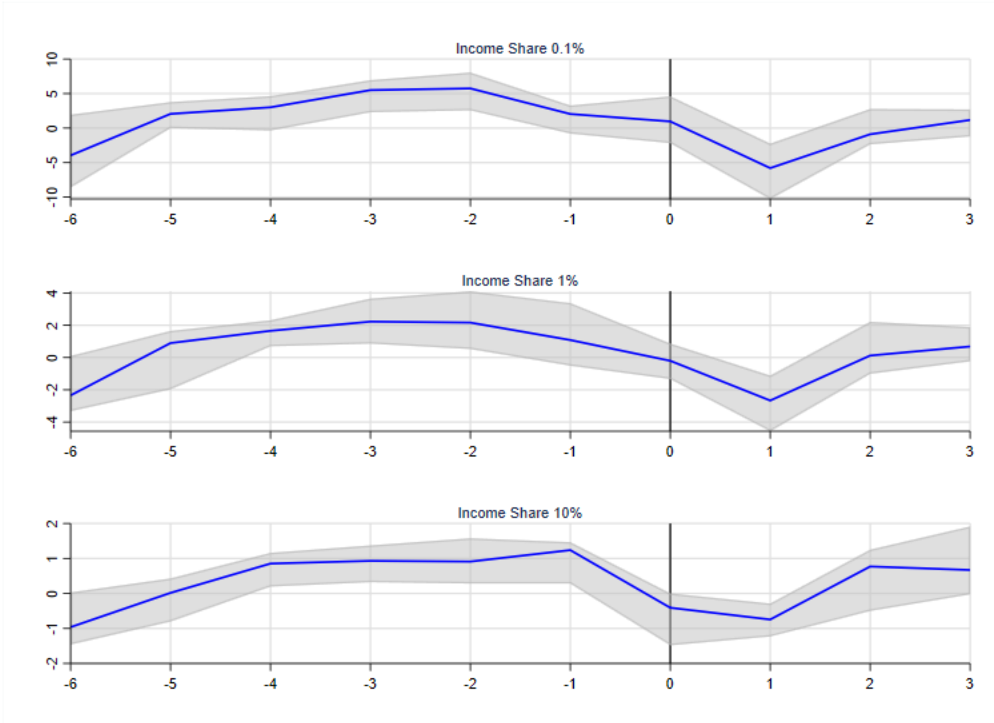
Figure 11: Share of Income Held by Top 0.1%, 1% and 10%.



Source: Author's estimation using Paul (2022)

share of the income held by the top percentiles of the income distribution behaves in the periods preceding a bank run. To do so, I estimate the median, the 33rd, and the 66th percentile of the annual percentage change seven periods before and after a given bank run in the sample. The results of this analysis are presented in Figure 12.

Figure 12: Median, 33rd and 66th Percentile of Income Shares



Source: Author estimation using the data from Paul (2022) and Baron et al. (2021).

The annual percentage changes in each income share measure increase drastically in the years before the bank run, followed by a drastic fall that lasts until after the bank run, and they start recovering after that. Furthermore, note the inverted U shape of the growth in the income inequality measures. This shape suggests that “rapid” inequality growth (i.e., changing from negative growth to positive growth from one period to another) precedes (or correlates with) an episode of bank runs. Finally, bank runs appear to reduce the income share held by the top percentiles of the income distribution (the 0. 1% and 1%), where the median reduction can go up to a 5% annual percentage change.

Second, I want to answer the following question. Is there a (positive or negative) correlation between income inequality and bank runs? Note that I am not implying any causality because, with the available data, it is impossible to discern any causal relationship between these two phenomena. The best thing that can be done with the available data is to establish the type of correlation that governs the relationship between income inequality and bank runs. To do so, let the probability of a bank run be described by

$$Pr\left(BR_{j,t}=1|\,\text{Ineq}_{j,t-1},X_{j,t-1};\boldsymbol{\beta}\right)=\frac{1}{1+\exp\left(-(\alpha_j+\beta_1\Delta_h\,\text{Ineq}_{j,t-1}+\beta_2\Delta_h X_{j,t-1}+\varepsilon_{j,t})\right)} \tag{55}$$

where α_j is country-specific constant, $\Delta_h\,\text{Ineq}_{j,t-1}$ is the change from period $t-1-h$ to $t-1$ of either measure of inequality or a vector that includes a combination of these measures for country j, $\Delta_h X_{j,t-1}$ is the change vector of controls X from $t-1-h$ to $t-1$, and $\varepsilon_{j,t}$ is the error term. Following Paul (2022), I normalize the variables in Ineq and X by their standard deviation. The selection of h is 4, following both Paul (2022) and Gorton and Ordoñez (2020).

The results of the estimate (55) are presented in Table 3. First, for the estimation of the results in columns (1), (5) and (9), I included the logarithmic change in the credit-to-GDP ratio as controls in addition to the country fixed effects. The first three columns are estimates for each inequality measure individually.[14] The table includes the point estimation of the odds ratio, the robust standard errors in parentheses, and the marginal effect in brackets for each explanatory variable.

The results in columns (1), (5) and (9) of Table 3 suggest that a standard deviation increase in the growth of the income share held by the top percentile is correlated with a 1 to 1.2 percentage point increase in the probability of a bank run. These percentage point increases in probability occur after controlling for the credit-to-GDP ratio, which has been deemed a determinant of the probability of financial crises in the literature (see Gorton and Ordoñez (2020), Paul (2022)). Recall that the unconditional probability of the bank run sample is 4%, so a percentage point increase is a fairly significant increase in the probability of bank runs.

Taking the previous results as benchmarks, additional robustness checks were performed to estimate the correlation between income inequality and bank runs. The robustness analysis follows that in Paul (2022) and is presented in the remaining columns of Table 3. Columns (2), (6), and (10) present the results, using as additional controls in addition to the 4-year change of credit-to-GDP ratio the following (in 4-year changes): investment-to-GDP ratio, public debt-to-GDP ratio, current account-to-GDP ratio, consumer price index, long- and short-term interest rates. The results suggest that even accounting for macroeconomic variables, an increased inequality is correlated with an increased probability of a bank run of around one percentage point.

[14] Including any combination of income share in one estimation will produce a high correlation between explanatory variables, generating biased point estimates.

Table 3: Probability of Bank Runs

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\Delta_4 \log \text{Credit}_{t-1}$	0.559***	0.516**	0.648***	0.420**	0.560***	0.553**	0.650***	0.413**	0.358**	0.399*	0.504**	0.372*
	(0.146)	(0.232)	(0.231)	(0.207)	(0.129)	(0.220)	(0.212)	(0.195)	(0.163)	(0.238)	(0.231)	(0.212)
	[0.015]	[0.008]	[0.010]	[0.004]	[0.015]	[0.008]	[0.009]	[0.004]	[0.010]	[0.005]	[0.006]	[0.003]
Δ_4Income Share 0.1%$_{t-1}$	0.393***	0.578***	0.618***	0.590***								
	(0.120)	(0.181)	(0.178)	(0.201)								
	[0.010]	[0.009]	[0.009]	[0.005]								
Δ_4Income Share 1%$_{t-1}$					0.443***	0.650***	0.701***	0.517**				
					(0.155)	(0.223)	(0.204)	(0.234)				
					[0.012]	[0.009]	[0.009]	[0.005]				
Δ_4Income Share 10%$_{t-1}$									0.345*	0.585***	0.576***	0.454**
									(0.178)	(0.208)	(0.214)	(0.214)
									[0.009]	[0.008]	[0.007]	[0.004]
Set of controls		✓	✓	✓		✓	✓	✓		✓	✓	✓
GDP & Global GDP			✓	✓			✓	✓			✓	✓
Stock & House Prices				✓				✓				✓
Number of crises	28	27	27	25	34	33	33	30	26	26	26	26
Observations	808	758	758	690	952	891	891	841	812	768	768	754
Countries	15	15	15	15	17	17	17	17	17	17	17	17
Country FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p-value	0.337	0.484	0.377	0.861	0.380	0.643	0.606	0.620	0.283	0.531	0.494	0.731
Pseudo R^2	0.067	0.162	0.187	0.279	0.073	0.179	0.203	0.281	0.040	0.172	0.195	0.276

Source: Author's elaboration using Paul (2022) and Baron et al. (2021) data.

In columns (3), (7), and (11), the estimation also controls for changes in the domestic and global real GDP. The results follow a trend similar to that in columns (2), (6), and (8): Their significance level increases and the point estimate of the marginal effects is smaller than that in the benchmark case. Finally, columns (4), (8) and (12) add real stock and house prices as additional controls. Note that its significance level remains relatively high for the change in the income share held by the top 0.1%. The significance level was reduced for the other two measures. At the same time, the point estimates of the marginal effects are almost identical between income shares, but smaller than those from the benchmark case.

In conclusion, increasing income inequality, in the form of an increase in the income share held by the top percentiles of the income distribution, correlates with an increased probability of bank runs. This correlation suggests that an increase in one standard deviation in the growth of such shares is roughly correlated with an increase of one percentage point in the probability of a bank run. These results strongly motivate the study of the mechanism underpinning such a correlation in a theoretical model.

6 Conclusion

In conclusion, this article provides evidence of a positive correlation between income inequality and the probability of bank run. The proposed banking model shows that increasing income inequality increases the probability of a bank run. In this sense, the model establishes a rationale between income inequality and financial instability. The widening gap between the rich and the poor has caused a shift in both the level of risk aversion and the amount of money people are willing to spend, making it difficult for banks to carry out their duties and leading to financial instability.

The findings of the paper have important implications for policymakers, suggesting that reducing income inequality can help prevent financial crises. In particular, policies that aim to redistribute wealth and reduce the concentration of wealth at the top can help stabilize the financial system and reduce the likelihood of bank runs. Moreover, the banking model provided in the article provides a framework for analyzing the effects of different policy interventions on the probability of a bank run.

Econometric analysis suggests that an increase in inequality by one standard deviation is associated with a 0.3 to 0.7 percentage point increase in the probability of a bank run. This result accounts for different covariates, such as GDP per capita, inflation, and financial development.

In general, the paper contributes to the growing literature on the relationship between income inequality and financial stability and highlights the need for policymakers to consider the distribution of wealth when designing policies to prevent financial crises.

Appendix

A Agents preferences and Relative Risk Aversion

The following utility function represents Hyperbolic Absolute Risk Aversion (HARA) preferences

$$U(c) = \frac{(c-\psi)^{1-\gamma}}{1-\gamma} \tag{56}$$

It follows that

$$U'(c) = (c-\psi)^{-\gamma} \tag{57}$$

$$U''(c) = -\gamma (c-\psi)^{-\gamma-1} \tag{58}$$

To satisfy the conditions described before for the problem, the second derivative must comply with

$$U''(c) < 0 \Leftrightarrow \begin{cases} \gamma > 1 \wedge c > 0 \wedge c > \psi \\ \frac{c_1}{2} \in \mathbb{Z} \wedge c_1 \leq -2 \wedge c_1 = -1-\gamma \wedge c > 0 \wedge \psi < -c \end{cases} \tag{59}$$

The Relative Risk Aversion is given by:

$$RRA = -c\frac{U''(c)}{U'(c)} = -\frac{c(-\gamma)(c-\psi)^{-\gamma-1}}{(c-\psi)^{-\gamma}} = \frac{\gamma c}{c-\psi} \tag{60}$$

Given that $\gamma > 0$ the utility will display Increasing, Constant, or Decreasing Relative Risk Aversion if:

$$\frac{\partial RRA}{\partial c} = \frac{-\psi\gamma}{(c-\psi)^2} \Rightarrow \begin{cases} \frac{\partial RRA}{\partial c} > 0 \Leftrightarrow \psi < 0 & \text{Increasing - IRRA} \\ \frac{\partial RRA}{\partial c} = 0 \Leftrightarrow \psi = 0 & \text{Constant - CRRA} \\ \frac{\partial RRA}{\partial c} < 0 \Leftrightarrow \psi > 0 & \text{Decreasing - DRRA} \end{cases} \tag{61}$$

If $\psi < 0$, the depositor still has some utility even if the consumption allocation is 0. In contrast, if $\psi > 0$, the depositor faces a decrease in its utility level for any consumption allocation greater than 0. The case of $\psi = 0$ is the typical utility of constant relative risk aversion (CRRA).

B Proofs

B.1 Proof of Proposition 2.1

The proposition is that a deposit contract should not lead to a run equilibrium in both states $s = H, L$ for both groups of agents $i = 1, 2$. The intuition is that if a contract leads to a run in both states, it can be improved upon by reallocating resources in a way that makes one group strictly better off without making the other worse off. That is, there exists a Pareto optimal allocation that dominates the allocations in this equilibrium.

Note that the type of the agent (i.e., early type or late type) is not revealed until $t = 1$. Thus, the bank should offer a deposit contract characterized by d_i for $i = 1, 2$ in period $t = 1$ which leads to a run of both groups in both states $s = H, L$. This contract should be such that it implies that all agents will not be able to recover the promised payment in $t = 2$, regardless of the state s, causing them to withdraw early.

The bank optimally chooses to invest all its resources in liquid assets y. That is, $y = \lambda\omega$ making that $c_{2is} = 0$ for $i = 1, 2$ and $s = H, L$. Now assume that the bank provides a fraction $a \in [0, 1]$ of ω to group 1 and $1 - a$ to group 2, and assume that $a < \frac{1}{2}$. This would suggest that group 2 would receive more in this equilibrium. In this sense, group 1 would have incentives to deviate from the intended equilibrium. Similarly, if $a > \frac{1}{2}$, group 1 would receive more than group 2, and this last group would have incentives to deviate. Thus, the only allocation that would not generate any deviations is the one with $a = \frac{1}{2}$. Hence, the bank's utility under this contract is characterized by

$$W = 2U\left(\frac{\omega}{2}\right) \tag{62}$$

Now, suppose the following deposit contract:

$$d_i = \frac{\omega}{2} \text{ for } i = 1, 2 \tag{63}$$

$$y = \lambda\omega \tag{64}$$

Given that $\lambda \in (0, 1)$, then the amount invested in the illiquid asset is $x = \omega - \lambda\omega = \omega\,(1 - \omega)$. The return on this asset is given by $R_H > R_L > 1$. Thus, the amount available to distribute among depositors at $t = 2$ is $R_s\omega\,(1 - \lambda)$. The deposit contract for $t = 2$ is of the form (assuming an equal distribution among groups) as follows:

$$c_{2is} = \frac{R_s\omega\,(1 - \lambda)}{2\,(1 - \lambda)} = \frac{R_s\omega}{2} \text{ for } i = 1, 2, s = H, L \tag{65}$$

Then, this last contract will dominate the original contract whenever

$$c_{2is} \geq \frac{\omega}{2} \Leftrightarrow \frac{R_s\omega}{2} \geq \frac{\omega}{2} \Leftrightarrow R_s \geq 1 \text{ for } i = 1, 2, s = H, L \tag{66}$$

It follows that since $R_H > R_L > 1$, the last inequality in the previous condition will hold with strict inequality. Since it holds for any agent i and any state s, the bank will never find the optimal way to offer a contract that induces a run in both states $s = H, L$, for both groups of agents. $\square$

B.2 Proof of Proposition 2.2

Suppose that the banks offer a contract in which some type of agent i run in the high state but do not run in the low state that satisfies

$$c_{2iL} \geq d_i > c_{2iH} \tag{67}$$

where d_i is the face value of the deposit contract that induces the supposed behavior of group i. Given that there is no run in the state $s = L$ by the $(1-\lambda)$ proportion of late-type agents of the group i, it must be true that $y \in (0,1)$ to invest in the illiquid asset $x = \omega - y > 0$ so as to have a $R_L(\omega - y)$ units to distribute among depositors conditional on being in state L. Now, suppose a contract in the form of

$$\begin{cases} \tilde{c}_{2iH} = c_{2iL} & \text{if } s = H \\ \tilde{c}_{2iL} = c_{2iL} & \text{if } s = L \end{cases} \tag{68}$$

Under this contract, the $(1-\lambda)$ agents of late-type in group i are at least as good as they were in the previous contract. On the other hand, the deposit contract that the bank has provided at $t = 2$ in this state H for both group i and j is given by

$$(1-\lambda)\left(\tilde{c}_{2iH} + \tilde{c}_{2jH}\right) = R_L(\omega - y) \text{ for } j \neq i \tag{69}$$

Note, however, that the return in state H is $R_H > R_L$, such that the bank will have an excedent of returns in $t = 2$ given by

$$(R_H - R_L)(\omega - y) > 0 \tag{70}$$

Hence, it is still possible to offer a welfare-improving allocation in which there is no run in the high state, and it does not necessarily lead to a run in the low state. □

B.3 Proof of Proposition 2.3

Suppose that the bank offers a contract that leads to a run for the group i in both states $s = H, L$. Note that the present contract could not lead to a run in both states for both groups (see Proposition 2.1) or that it leads to a run by group $j \neq i$ in state H but not in state L (see Proposition 2.2). It follows that the only two remaining scenarios are that this contract does (not) lead to a run in state L for group j, but never in state H.

First, assume that the present contract does not lead to a run for the group j in either state. This means that

$$\begin{cases} c_{2js} \geq d_j & \text{for states } s = H, L \\ d_i > c_{2is} & \text{for states } s = H, L \end{cases} \tag{71}$$

The bank has to invest in the liquid asset at least the amount to cover the deposit contracts of the group i that ran on the bank and the proportion λ of early types of group j. That is,

$$y \geq d_i + \lambda d_j > \lambda d_i + \lambda d_j \tag{72}$$

The second inequality characterizes the total face value of a contract that does not lead to a run for group i in either state and given that $\lambda \in (0,1)$. Under the present contract, given that $y \in (0, \omega)$ since it needs to be large enough to provide non-zero consumption at $t = 1$ for either group and it needs to invest some in the illiquid asset since $R_s > 1$ in either state $s = H, L$. Initially, assume that $y > d_i + \lambda d_j$, then the total amount that the bank would have to distribute at $t = 2$ is given by

$$R_s (\omega - y) + y - d_i - \lambda d_j \text{ for } s = H, L \tag{73}$$

Now, suppose that the bank offers a contract such that group i does not lead to a run in state $s = H$ but leads to a run in state L (given proposition 2.1 and 2.2). Furthermore, assume that the contract is such that $\varrho d_i + \lambda d_j < y$ where $\varrho \in (0,1)$and $\varrho \leq \lambda$ is the proportion of agents that do not run on the bank of group i. This contract will dominate the original contract because, for any state $s = H, L$, the total amount that the bank can distribute will be greater. That is,

$$R_s (\omega - y) + y - d_i - \lambda d_j < R_s (\omega - y) + y - \varrho d_i - \lambda d_j \leq R_s (\omega - y) + y - \lambda d_i - \lambda d_j \tag{74}$$

Hence, the depositors will have deposit contracts at least as big as the original contract. Note that in the case of $y = d_i + \lambda d_j$, the pie in $t = 2$ in either state is $R_s (\omega - y)$ and it will be strictly smaller than $R_s (\omega - y) + y - d_i - \lambda d_j$. Consequently, the previous result holds for $y \geq d_i + \lambda d_j$.

Second, assume that the bank offers a contract that leads to a run in state L for group j, then the bank would have to invest in the liquid asset

$$y \geq d_i + d_j \text{ for } j \neq i \tag{75}$$

Note that this case will be dominated by the case where the bank offers a contract that does not lead to a run in state L for group j, given that

$$y \geq d_i + d_j > d_i + \lambda d_j > \lambda d_i + \lambda d_j \tag{76}$$

Consequently, it is never optimal to choose a contract that leads to a run in **BOTH** states $s = H, L$ by some group i. $\square$

B.4 Proof of Proposition 2.4

Suppose that it does not hold with equality. That is, $\lambda(d_1 + d_2) < y$. This suggests that some of the investment in the liquid asset is left over on date 1. The bank could reduce the amount invested in such an asset by $\epsilon > 0$ and invest in the illiquid asset with a return $R_s > 1$ for all $s = H, L$. The net change in the available return for consumption at $t = 2$ is $(R - 1)\,\epsilon > 0$. Hence, one could improve the consumption of the late-type consumer without affecting the consumption of the early-type. This cannot be optimal. It follows that in any optimal plan $\lambda(d_1 + d_2) = y$. $\square$

C Log-Linearization of the Euler equation in Case 1

Let $z_t = c_t - \psi$, so the Euler conditions in (25) are

$$z_t^{-\gamma} = \beta \left(\pi_H R_H z_{t+1,H}^{-\gamma} + \pi_H R_L z_{t+1,L}^{-\gamma}\right) = \beta E\left[R z_{t+1}^{-\gamma}\right] \tag{77}$$

Taking logs,

$$0 = \log(\beta) + E\left[\log(R)\right] - E\left[\gamma \log(z_{t+1})\right] + \gamma \log(z_t) \tag{78}$$

Now, doing a first order Taylor Expansion around z^*,

$$-\gamma \log(z^*) - \frac{\gamma}{z^*}(z_t - z^*) = \log(\beta) + E\left[\log(R)\right] - E\left[\gamma \log(z^*)\right] - E\left[\frac{\gamma}{z^*}(z_{t+1} - z^*)\right] \tag{79}$$

Reorganizing terms

$$-\log(\beta) - E\left[\log(R)\right] = E[(z_{t+1} - z_t)] \tag{80}$$

Replacing z_t and substituting the consumption allocations for each group i, the solution for y_i^* is given by,

$$y_1^* = \frac{\lambda \bar{R} \omega}{\lambda \bar{R} + (1-\lambda)} + \frac{\lambda(1-\lambda)(1+A)}{\left(\lambda \bar{R} + (1-\lambda)\right)}\left(\log(\beta) + E\left[\log(R)\right]\right) \tag{81}$$

$$y_2^* = \frac{\lambda \bar{R} \omega}{\lambda \bar{R} + (1-\lambda)} + \frac{\lambda(1-\lambda)(1+A)}{A\left(\lambda \bar{R} + (1-\lambda)\right)}\left(\log(\beta) + E\left[\log(R)\right]\right) \tag{82}$$

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